

Spin-plasmons in topological insulator

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Collective plasmon excitations in a helical electron liquid on the surface of strong three-dimensional topological insulator are considered. The properties and internal structure of these excitations are studied. Due to spin-momentum locking in helical liquid on a surface of topological insulator, the collective excitations should manifest themselves as coupled charge- and spin-density waves.

I. INTRODUCTION

In recent years, topological insulators with a non-trivial topological order, intrinsic to their band structure, were predicted theoretically and observed experimentally (see [1] and references therein). Three-dimensional (3D) realizations of “strong” topological insulators (such as Bi₂Se₃, Bi₂Te₃ and Sb₂Te₃) are insulating in the bulk, but have gapless topologically protected surface states with a number of unusual properties [2]. These states obey two-dimensional Dirac equation for massless particles, similar to that for electrons in graphene [3], but related to real spin of electrons, instead of sublattice pseudospin in graphene.

The consequence of that is a spin-momentum locking for electrons on the surface of strong 3D topological insulators, i.e. spin of each electron is always directed in the surface plane and perpendicularly to its momentum [1, 4]. The surface of topological insulator can be chemically doped, forming a charged “helical liquid”.

Collective excitation of electrons in such helical liquid were considered in [5], where relationships between charge and spin responses to electromagnetic field were derived. It was shown that charge-density wave in this system is accompanied by spin-density wave. Application of spin-plasmons to create “spin accumulator” was proposed in [6]. Also the surface plasmon-polaritons under conditions of magnetoelectric effect in 3D topological insulator were considered [7].

In the present article we consider the properties and internal structure of spin-plasmons in a helical liquid. Within the random-phase approximation, we derive plasmon wave function and calculate amplitudes of charge- and spin-density waves in the plasmon state.

II. WAVE FUNCTION OF SPIN-PLASMON

Low-energy effective Hamiltonian of the surface states of Bi₂Se₃ in the representation of spin states $\{|\uparrow\rangle, |\downarrow\rangle\}$ is $H_0 = v_F(p_x\sigma_y - p_y\sigma_x)$ for a surface in the xy plane,

where the Fermi velocity $v_F \approx 6.2 \times 10^5$ m/s [2]. Its eigenfunctions can be written as $e^{i\mathbf{p}\cdot\mathbf{r}}|f_{\mathbf{p}\gamma}\rangle/\sqrt{S}$, where S is the system area and $|f_{\mathbf{p}\gamma}\rangle = (e^{-i\varphi_{\mathbf{p}}/2}, i\gamma e^{i\varphi_{\mathbf{p}}/2})^T/\sqrt{2}$ is the spinor part of the eigenfunction, corresponding to electron with momentum \mathbf{p} (its azimuthal angle in the xy plane is $\varphi_{\mathbf{p}}$) from conduction ($\gamma = -1$) or valence ($\gamma = +1$) band. Many-body Hamiltonian of electrons populating the surface of topological insulator is $H = \sum_{\mathbf{p}\gamma} \xi_{\mathbf{p}\gamma} a_{\mathbf{p}\gamma}^\dagger a_{\mathbf{p}\gamma} + (1/2S) \sum_{\mathbf{q}} V_{\mathbf{q}} \rho_{\mathbf{q}}^\dagger \rho_{\mathbf{q}}$, where $a_{\mathbf{p}\gamma}$ is the destruction operator for electron with momentum \mathbf{p} from the band γ , $\xi_{\mathbf{p}\gamma} = \gamma v_F |\mathbf{p}| - \mu$ is its energy measured from the chemical potential μ , $V_{\mathbf{q}} = 2\pi e^2/\varepsilon q$ is the Coulomb interaction; $\rho_{\mathbf{q}}^\dagger = \sum_{\mathbf{p}\gamma\gamma'} \langle f_{\mathbf{p}+\mathbf{q},\gamma'} | f_{\mathbf{p}\gamma} \rangle a_{\mathbf{p}+\mathbf{q},\gamma'}^\dagger a_{\mathbf{p}\gamma}$ is the charge density operator for helical liquid.

The creation operator for spin-plasmon with wave vector \mathbf{q} can be presented in the form:

$$Q_{\mathbf{q}}^+ = \sum_{\vec{p}\gamma\gamma'} C_{\mathbf{p}\mathbf{q}}^{\gamma'\gamma} a_{\mathbf{p}+\mathbf{q},\gamma'}^\dagger a_{\mathbf{p}\gamma}. \quad (1)$$

This operator should obey the equation of motion $[H, Q_{\mathbf{q}}^+] = \Omega_{\mathbf{q}} Q_{\mathbf{q}}^+$, where $\Omega_{\mathbf{q}}$ is the plasmon frequency. We can get solution of this equation in the random phase approximation at $T = 0$ (similarly to [8]):

$$C_{\mathbf{p}\mathbf{q}}^{\gamma'\gamma} = \frac{|n_{\mathbf{p}\gamma} - n_{\mathbf{p}+\mathbf{q},\gamma'}| \langle f_{\mathbf{p}+\mathbf{q},\gamma'} | f_{\mathbf{p}\gamma} \rangle N_{\mathbf{q}}}{\Omega_{\mathbf{q}} + \xi_{\mathbf{p}\gamma} - \xi_{\mathbf{p}+\mathbf{q},\gamma'} + i\delta}, \quad (2)$$

where $n_{\mathbf{p}+} = \Theta(p_F - |\mathbf{p}|)$ and $n_{\mathbf{p}-} = 1$ are occupation numbers for electron-doped helical liquid ($p_F = \mu/v_F$ is the Fermi momentum).

The plasmon frequency is determined in this approach from the equation $1 - V_{\mathbf{q}}\Pi(q, \Omega_{\mathbf{q}}) = 0$, where

$$\Pi(q, \omega) = \frac{1}{S} \sum_{\mathbf{p}\gamma\gamma'} \frac{|\langle f_{\mathbf{p}+\mathbf{q},\gamma'} | f_{\mathbf{p}\gamma} \rangle|^2 (n_{\mathbf{p}\gamma} - n_{\mathbf{p}+\mathbf{q},\gamma'})}{\omega + \xi_{\mathbf{p}\gamma} - \xi_{\mathbf{p}+\mathbf{q},\gamma'} + i\delta} \quad (3)$$

is the polarization operator of the helical liquid, different from that for graphene [3] only by degeneracy factor. The factor $N_{\mathbf{q}}$ in (2) can be determined from the normalization condition

$$\begin{aligned} \langle 0 | [Q_{\mathbf{q}}, Q_{\mathbf{q}'}^\dagger] | 0 \rangle &= \delta_{\mathbf{q}\mathbf{q}'} \sum_{\gamma\gamma'} D_{\gamma'\gamma} = \delta_{\mathbf{q}\mathbf{q}'}, \\ D_{\gamma'\gamma} &= \sum_{\mathbf{p}} |C_{\mathbf{p}\mathbf{q}}^{\gamma'\gamma}|^2 (n_{\mathbf{p}\gamma} - n_{\mathbf{p}+\mathbf{q},\gamma'}), \end{aligned} \quad (4)$$

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($|0\rangle$ is the ground state), so that $|N_{\mathbf{q}}|^{-2} = -S[\partial\Pi(q, \omega)/\partial\omega]|_{\omega=\Omega_q}$. The quantities $D_{\gamma'\gamma}$ in (4) can be considered as total weights of intraband (D_{++}) and interband ($D_{+-} + D_{-+} = 1 - D_{++}$) electron transitions, contributing to the plasmon wave function (1). Note that all these formulas are also applicable to the case of graphene.

Spin-plasmon dispersion Ω_q and contribution of intraband transitions into its wave function are plotted in Fig. 1 at various $r_s = e^2/\varepsilon v_F$, where ε is the dielectric susceptibility of surrounding 3D medium. For Bi_2Se_3 , $r_s \approx 0.09$ with $\varepsilon \approx 40$ for dielectric half-space [5] (for such small r_s , the corresponding dispersion curve approaches very closely to the upper bound $\omega = v_F q$ of the intraband continuum). The results for suspended graphene with rather large $r_s = 8.8$ (for $v_F \approx 10^6$ m/s, $\varepsilon = 1$ and with the degeneracy factor 4 incorporated into r_s) are also presented for comparison. It is seen that the undamped spin-plasmon consists mainly of intraband transitions. When the dispersion curve enters the interband continuum, the spin plasmon becomes damped and inter- and intraband transitions contribute almost equally to its wave function.

III. CHARGE- AND SPIN-DENSITY WAVES

The helical liquid in the state $|1_{\mathbf{q}}\rangle = Q_{\mathbf{q}}^+|0\rangle$ with one spin-plasmon of wave vector \mathbf{q} has a distribution of electron-hole excitations (2), shifted towards \mathbf{q} . Due to the spin-momentum locking, the system acquires a total nonzero spin polarization, perpendicular to \mathbf{q} . A similar situation occurs in the current-carrying state of the helical liquid, which turns out to be spin-polarized [4].

Introducing one-particle spin operator as $\mathbf{s} = \boldsymbol{\sigma}/2$, we can calculate its average value in the one-plasmon state $\langle \mathbf{s} \rangle = \langle 1_{\mathbf{q}} | \mathbf{s} | 1_{\mathbf{q}} \rangle$ as

$$\langle \mathbf{s} \rangle = \sum_{\mathbf{p}\gamma\gamma'\tau} \left[\langle f_{\mathbf{p}+\mathbf{q},\gamma'} | \mathbf{s} | f_{\mathbf{p}+\mathbf{q},\tau} \rangle C_{\mathbf{p}\mathbf{q}}^{\tau\gamma} - C_{\mathbf{p}\mathbf{q}}^{\gamma'\tau} \langle f_{\mathbf{p}\tau} | \mathbf{s} | f_{\mathbf{p}\gamma} \rangle \right] \times \left(C_{\mathbf{p}\mathbf{q}}^{\gamma'\gamma} \right)^* (n_{\mathbf{p}\gamma} - n_{\mathbf{p}+\mathbf{q},\gamma'}). \quad (5)$$

If \mathbf{q} is parallel to \mathbf{e}_x , only the y -component of $\langle \mathbf{s} \rangle$ is nonzero. Its dependence on q at various r_s is plotted in Fig. 2(a).

Charge- and spin-density waves, accompanying spin-plasmon with the wave vector \mathbf{q} , can be characterized by corresponding spatial harmonics of charge- and spin-density operators: $\rho_{\mathbf{q}}^+$ and $\mathbf{s}_{\mathbf{q}}^+$ = $\sum_{\mathbf{p}\gamma\gamma'} \langle f_{\mathbf{p}+\mathbf{q},\gamma'} | \mathbf{s} | f_{\mathbf{p}\gamma} \rangle a_{\mathbf{p}+\mathbf{q},\gamma'}^\dagger a_{\mathbf{p}\gamma}$. Using, similarly to [9], the unitary transformation, inverse with respect to (1), we can write: $\rho_{\mathbf{q}}^+ = S N_{\mathbf{q}}^* \Pi^*(q, \Omega_q) Q_{\mathbf{q}}^+ + \tilde{\rho}_{\mathbf{q}}^+$ and $\mathbf{s}_{\mathbf{q}}^+ = S N_{\mathbf{q}}^* \boldsymbol{\Pi}_s^*(q, \Omega_q) Q_{\mathbf{q}}^+ + \tilde{\mathbf{s}}_{\mathbf{q}}^+$, where the operators $\tilde{\rho}_{\mathbf{q}}^+$ and $\tilde{\mathbf{s}}_{\mathbf{q}}^+$ are the contributions of single-particle excitations and are dynamically independent on plasmons. Here the crossed spin-density susceptibility of the helical liquid [5] has

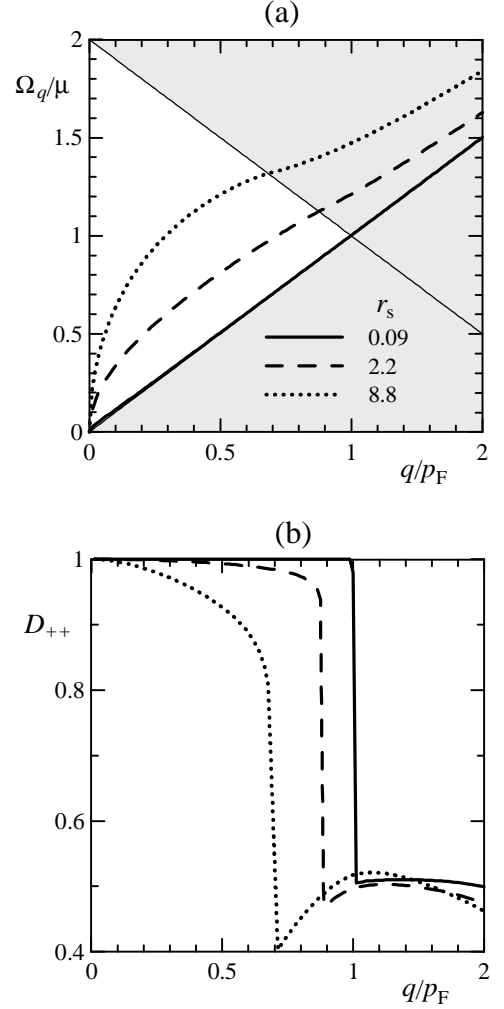


FIG. 1: Dispersions of spin-plasmon (a) and contributions D_{++} of intraband transitions into its wave function (b) at various r_s . Continuums of intraband ($\omega < v_F q$) and interband ($\omega + v_F q > 2\mu$) single-particle excitations are shaded in (a).

been introduced:

$$\Pi_s(q, \omega) = \frac{1}{S} \sum_{\mathbf{p}\gamma\gamma'} \frac{n_{\mathbf{p}\gamma} - n_{\mathbf{p}+\mathbf{q},\gamma'}}{\omega + \xi_{\mathbf{p}\gamma} - \xi_{\mathbf{p}+\mathbf{q},\gamma'} + i\delta} \times \langle f_{\mathbf{p}+\mathbf{q},\gamma'} | f_{\mathbf{p}\gamma} \rangle \langle f_{\mathbf{p}\gamma} | \mathbf{s} | f_{\mathbf{p}+\mathbf{q},\gamma'} \rangle. \quad (6)$$

The average values of $\rho_{\mathbf{q}}^+$ and $\mathbf{s}_{\mathbf{q}}^+$ in the $n_{\mathbf{q}}$ -plasmon state $|n_{\mathbf{q}}\rangle = [(Q_{\mathbf{q}}^+)^{n_{\mathbf{q}}}/(n_{\mathbf{q}}!)^{-1/2}]|0\rangle$ vanish, therefore we consider their mean squares in $|n_{\mathbf{q}}\rangle$ after subtracting their background values in $|0\rangle$, i.e.

$$\langle \rho_{\mathbf{q}} \rho_{\mathbf{q}}^+ \rangle \equiv \langle n_{\mathbf{q}} | \rho_{\mathbf{q}} \rho_{\mathbf{q}}^+ | n_{\mathbf{q}} \rangle - \langle 0 | \rho_{\mathbf{q}} \rho_{\mathbf{q}}^+ | 0 \rangle = n_{\mathbf{q}} S^2 |N_{\mathbf{q}}^* \Pi(q, \Omega_q)|^2, \quad (7)$$

$$\langle s_{\mathbf{q}}^{\perp} (s_{\mathbf{q}}^{\perp})^+ \rangle \equiv \langle n_{\mathbf{q}} | s_{\mathbf{q}}^{\perp} (s_{\mathbf{q}}^{\perp})^+ | n_{\mathbf{q}} \rangle - \langle 0 | s_{\mathbf{q}}^{\perp} (s_{\mathbf{q}}^{\perp})^+ | 0 \rangle = n_{\mathbf{q}} S^2 |N_{\mathbf{q}}^* \Pi_s^{\perp}(q, \Omega_q)|^2 \quad (8)$$

(only the in-plane transverse component s^{\perp} of the spin \mathbf{s} is nonzero in these averages). The normalized

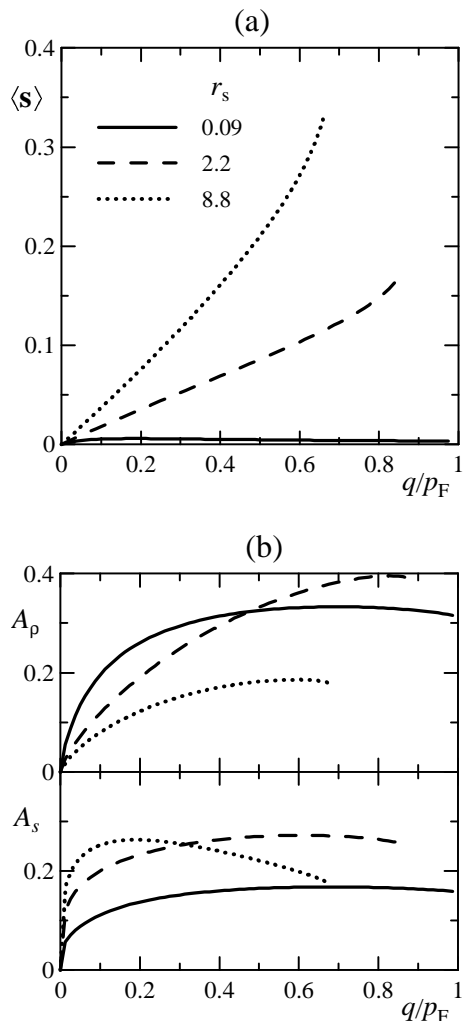


FIG. 2: Total spin polarization $\langle \mathbf{s} \rangle$ of the helical liquid in the one-plasmon state (a) at various r_s and normalized amplitudes A_ρ and A_s of charge- and spin-density waves respectively in the many-plasmon state (b).

amplitudes $A_\rho(q) = [\langle \rho_{\mathbf{q}} \rho_{\mathbf{q}}^\dagger \rangle / n_{\mathbf{q}} S \rho]^{1/2}$ and $A_s(q) = [\langle s_{\mathbf{q}}^\perp (s_{\mathbf{q}}^\perp)^\dagger \rangle / n_{\mathbf{q}} S \rho]^{1/2}$ of charge- and spin-density waves are plotted in Fig. 2(b) ($\rho = p_F^2 / 4\pi$ is the average electron density). The “continuity equation” for density and transverse spin, following from the spin-momentum locking [5], requires that $\Omega_q A_\rho(q) = 2v_F q A_s(q)$, in agreement with our results.

IV. CONCLUSIONS

We have considered microscopically spin-plasmons in helical liquid in the random phase approximation. The developed quantum-mechanical formalism can be applied for a number of problems in spin-plasmon optics.

We calculated the average spin polarization, acquired by the helical liquid in a spin-plasmon state, as well as mean-square amplitudes of charge- and spin-density waves, arising in this state. Coupling between these amplitudes, caused by spin-momentum locking, was demonstrated. The interconnection between charge- and spin density waves can be applied for constructing various spin-plasmonic and spintronic devices.

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- [1] M.Z. Hasan and C.L. Kane, Rev. Mod. Phys. **82**, 3045 (2010).
 - [2] H. Zhang, C.-X. Liu, X.-L. Qi, X. Dai, Z. Fang and S.-C. Zhang, Nature Phys. **5**, 438 (2009).
 - [3] A.H. Castro Neto, F. Guinea, N.M.R. Peres, K.S. Novoselov and A.K. Geim, Rev. Mod. Phys. **81**, 109 (2009).
 - [4] D. Culcer, E.H. Hwang, T.D. Stanescu and S. Das Sarma, Phys. Rev. B **82**, 155457 (2010).
 - [5] S. Raghu, S.B. Chung, X.L. Qim and S.-C. Zhang, Phys. Rev. Lett. **104**, 116401 (2010).
 - [6] I. Appelbaum, H.D. Drew and M.S. Fuhrer, Appl. Phys. Lett. **98**, 023103 (2011).
 - [7] A. Karch, arxiv:cond-mat/1104.4125v2.
 - [8] K. Sawada, K.A. Brueckner, N. Fukuda and R. Brout, Phys. Rev. **108** 507 (1957).
 - [9] R. Brout, Phys. Rev. **108**, 515 (1957).